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Dynamical Complexity of the Environmental Interfaces

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Abstract: Complex dynamics of the environmental interfaces, here considered to be the ground surface, has been modelled based on energy transfer across it. The continuous version of the model when discretized, provides a coupled set of difference equation of the involving environmental interface temperature and deep soil temperature. Numerical simulation of such a system based on various control parameters of the system enable us to quantify the complex behavior observed in such systems.

***Keywords**:* Complex system, Dynamics, Discrete system, Environment.

1. Introduction:

Some nonlinear systems internally composed of multi-components and during evolution do not follow any definite rule and evolve independently. Such nonlinear systems are termed as complex system, and collective response on evolution of internal components termed as, *complexity*, a property, of the complex system [1]. Thus, during evolution, a complex system may display chaos at some parameter space which further quantify the complexity of the system. Chaos is a state of the evolving nonlinear system displaying sensitivity to initial conditions and therefore predictability fails and disorder prevails. Lyapunov exponents or Lyapunov characteristic exponents (LCEs) is one of

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the measure of chaos such that if at any state LCEs > 0 then the system evolution is chaotic and, if LCEs < 0 the system evolution is regular, [2-3]. Further, another measure of complexity of a system is provided by the increase of its topological entropy. Increased fluctuation in topological entropy results in a more complex dynamic behavior observed in the system [4-5].

Weather and climate evolutions are most interesting and fascinating phenomena and studies on their perfect prediction is a great challenging task. While weather prediction is possible for short duration, for a few days, climate prediction may be possible for long duration, for years, for decades or longer [6-7]. One of the interesting area in climate modelling involves the dynamics of the environmental interfaces, considered here as ground surface [7]. It is mainly due to the fact the ground interface is involved in energy transfer across it due to incoming and outgoing radiation, heat convection, atmospheric moisture and heat conduction into deeper layers etc. In the present study, we consider various energy balance across the interface, which further mathematically provides a coupled system involving various parameters. Based on specific values of various involved parameter, the coupled dynamical exhibit complex behavior and patterns in phase space. Such complex dynamical behavior has been further quantified in terms of bifurcation scenario, Lyapunov exponents, attractors in phase space and topological entropies.

In section 2, we briefly outline the derivation of the coupled system involving the environmental interface temperature and the deeper soil layer temperature. In section 3, we provide results of numerical simulation and in section 4, the investigation is concluded.

2. Description of Dynamic Environmental/ Climate Model

In this section, we consider the energy balance equation resulting in a climate/ environmental model. Here energy transfer across an environmental interface involves the input radiation, and outgoing radiation and heat conduction in the ground interior as envisaged in [7]. Mathematically, briefly, one may consider the following set of equations describing the dynamics of the energy transfer processes [7]:

$$C_1 \frac{\Delta T_1}{\Delta t} = R_N - H - \Lambda E - G,$$

$$R_N = C_2(T_1 - T_2),$$

$$H = C_3(T_1 - T_2),$$

$$\Lambda E = C_4\, d\left[b(T_1 - T_2) + \frac{b^2}{2}(T_1 - T_2)^2\right],$$

$$G = C_5(T_1 - T_3),$$

where C_1, C_2, C_3, C_4 and C_5 refers to respectively the soil heat capacity, net radiation coefficient, heat transfer coefficient, water vapor coefficient and heat conduction coefficient. Further, T_1, T_2 and T_3 refers to ground surface temperature, air temperature at reference level deeper soil temperature respectively. In the above set of equation, R_N corresponds to net heat radiation, H the heat flux, ΛE the latent heat flux, G heat inside the soil and other parameters are having the same meaning [7].

Following the usual discretization of the derivatives, and defining

$$X = \frac{(T_1 - T_2)}{T_0};\, Y = \frac{(T_3 - T_2)}{T_0}$$

as the dimensionless interface temperature, the deeper soil temperature respectively and T_0 as mean earth temperature, the following coupled system is obtained:

$$X_{n+1} = AX_n(1 - X_n) + \frac{CB}{A}Y_n,$$

$$Y_{n+1} = \frac{DA}{B}X_n + (1 - D)Y_n. \tag{2.1}$$

Here, A, B, C, D as given in [7], are parameters of the system. The parameter A lies in the range $0 \le A \le 4$ and other parameters B, C, D, ranged in interval [0,1]. Replacing A by r, $\frac{CB}{A}$ by C, $\frac{DA}{B}$ by b and $(1 – D)$ by k, equation (2.1) can be re-written as

$$X_{n+1} = rX_n(1 - X_n) + cY_n$$

$$Y_{n+1} = bX_n + ky_n \tag{2.2}$$

Parameters c, b, k, considered here different in general but in certain circumstances they may be considered equal [7]. Fixed points of system (2.2) are obtained as

$$P_1^*(0,0) \text{ and } P_2^*\left(\frac{1-bc-r-k(1-r)}{r(k-1)}, \frac{-b(1-bc-k-r+kr)}{r(k-1)^2}\right) \tag{2.3}$$

Jacobian matrix of system (2.2) obtained as

$$J = \begin{pmatrix} r(1-x) - rx & c \\ b & k \end{pmatrix} \tag{2.4}$$

Eigenvalues corresponding to fixed point $P_1^*(0,0)$ are given by

$$\lambda_{1,2} = \frac{1}{2}\left(k + r \pm \sqrt{k^2 + r^2 + 4bc - 2kr}\right) \tag{2.5}$$

Thus, the stability criteria of $P_1^*(0,0)$ depends on the values of parameters *r*, *b*, *c* and *k* and so, once these parameters assigned numerical values stability criteria of $P_1^*(0,0)$ be established. In similar ways, for assigned values of parameters *r*, *b*, *c* and *k*, stability criteria of

$$P_2^*\left(\frac{1-bc-r-k(1-r)}{r(k-1)}, \frac{-b(1-bc-k-r+kr)}{r(k-1)^2}\right)$$

can be established.

For example, if we take $r = 2.5, b = 1.5, c = 0.1, k = 0.05$, we obtain fixed points as $P_1^*(0,0)$ and $P_2^*(0.663,1.04)$. Then, by using stability analysis we get $P_1^*(0,0)$ is unstable while $P_2^*(0.663,1.04)$ is stable.

3. Numerical Simulations of the Coupled System

In the following, we present the results of numerical simulation of the coupled system, eq. (2.2) both graphically and numerically.

3a. Bifurcations Phenomena:

Bifurcation diagrams of system (2.2) are drowned along *x* and *y* axes, first by varying parameters *r*, $2.3 \leq r \leq 3.5$ while fixing parameters $b = 1.5, c = 0.1, k = 0.05$ and presented in Figure 1. Then, again bifurcation diagrams have been obtained first by varying parameters *b*, $0 \leq b \leq 3.5$ and fixing parameters $r = 3.0, c = 0.1, k = 0.05$ and presented in Figure 2. Lower figures in both, Figure 1 and Figure 2, are presenting bifurcation scenario within periodic windows, respectively, for $3.37 \leq r \leq 3.5$ and $2.9 \leq r \leq 3.15$.

Looking carefully these bifurcation diagrams one finds that the system initially started displaying period doubling scenario followed by chaos. Then, as in Figure 1, when parameter r allowed to vary, it displays periodic windows of period 5 which again splits into period doubling-chaos adding phenomena. Similar situations again appear in case when parameter b allowed to vary in Figure 2. Here, within periodic window one finds period 7 orbits which again splits into period doubling -chaos adding scenario. Deep analysis within different periodic windows may reveal phenomena like bi or multi-stability, intermittency etc.

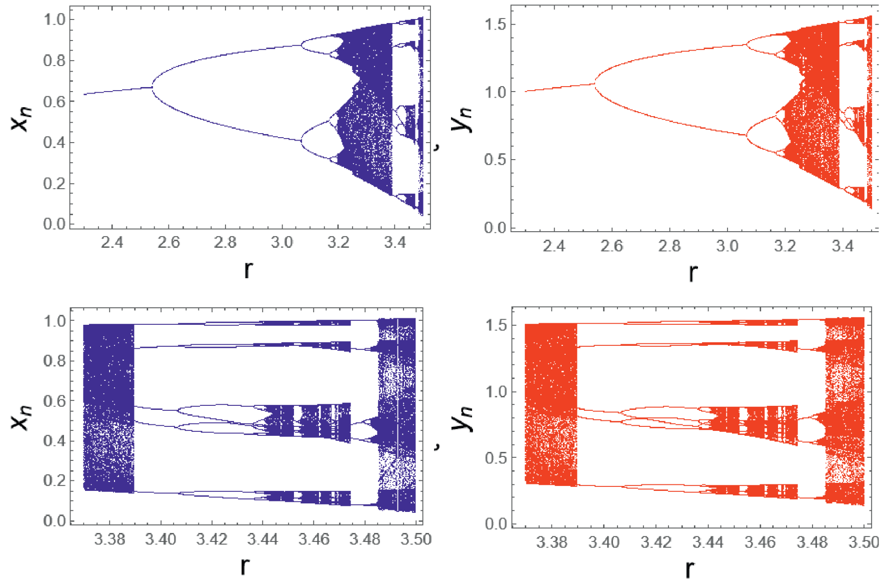

Figure 1: Bifurcation obtained for map (2.2) along X and Y directions for the values of the parameters b = 1.5, c = 0.1, k = 0.05 and then (i) 2.3 ≤ r ≤ 3.5, (upper row figures) and (ii) 3.37 ≤ r ≤ 3.5, (lower row figures), displaying scenario within a periodic windows.

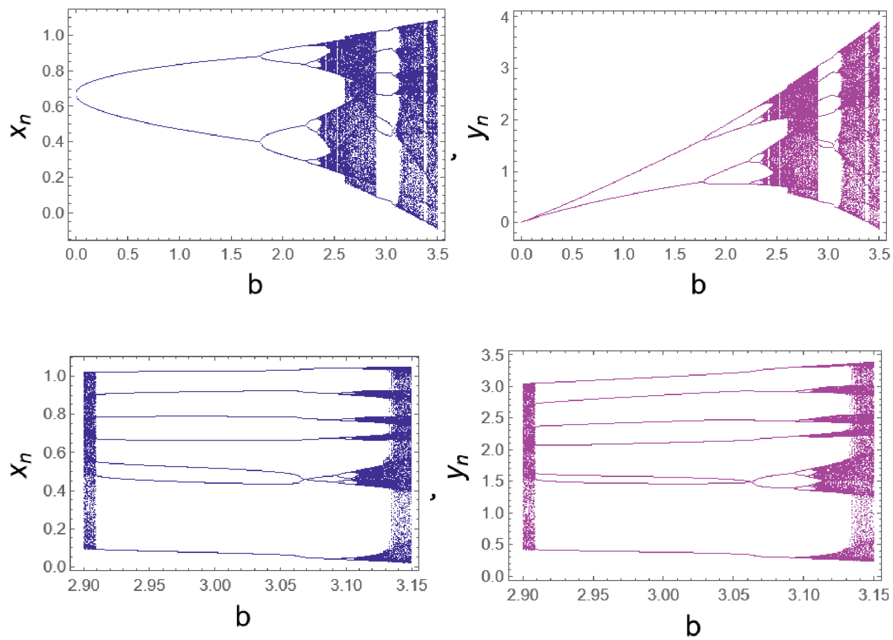

Figure 2: Bifurcation obtained for map (2.2) along x and y directions for the values of the parameters r = 3.0, c = 0.1, k = 0.05 and then, (i) 0 ≤ b ≤ 3.5, (upper row figures) and (ii) 2.9 ≤ r ≤ 3.15, (lower row figures), displaying scenario within a periodic windows.

3b. Regular and Chaotic Attractors and calculations of Lyapunov Exponents (LCEs)

Fixing values of parameters $b = 1.5$, $c = 0.1$, $k = 0.05$ and changing values of parameter *r*, regular and chaotic are obtained for system (2.2) and presented in Figure 3. Lyapunov exponents, (LCEs), are obtained for some cases of regular and chaotic motions of some cases and shown in Figure 4. As expected, LCEs are negative (LCE < 0) here for regular cases and are positive (LCE > 0) for chaotic cases. In Figure 5, regular and chaotic attractors are shown when parameter *b* takes different values and in Figure 6, plots of LCEs are presented for some cases here.

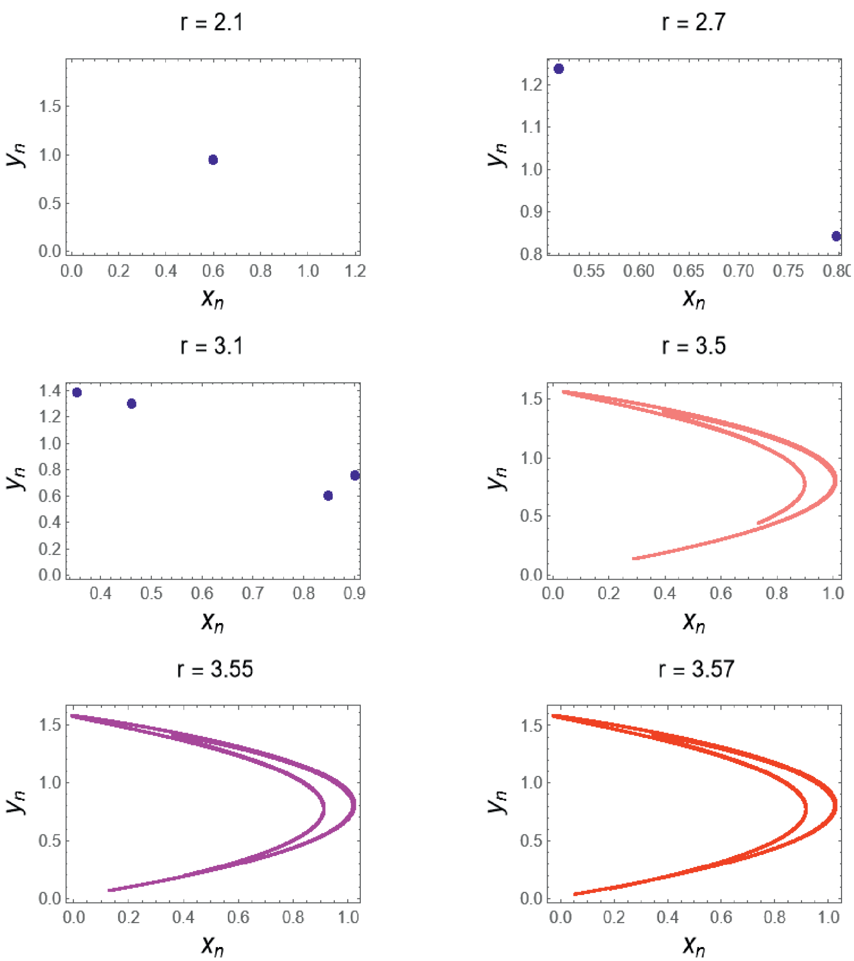

Figure 3: Plots of regular and chaotic attractors of system (2.1) for different values of parameter *r* when other parameters are *b* = 1.5, *c* = 0.1, *k* = 0.05

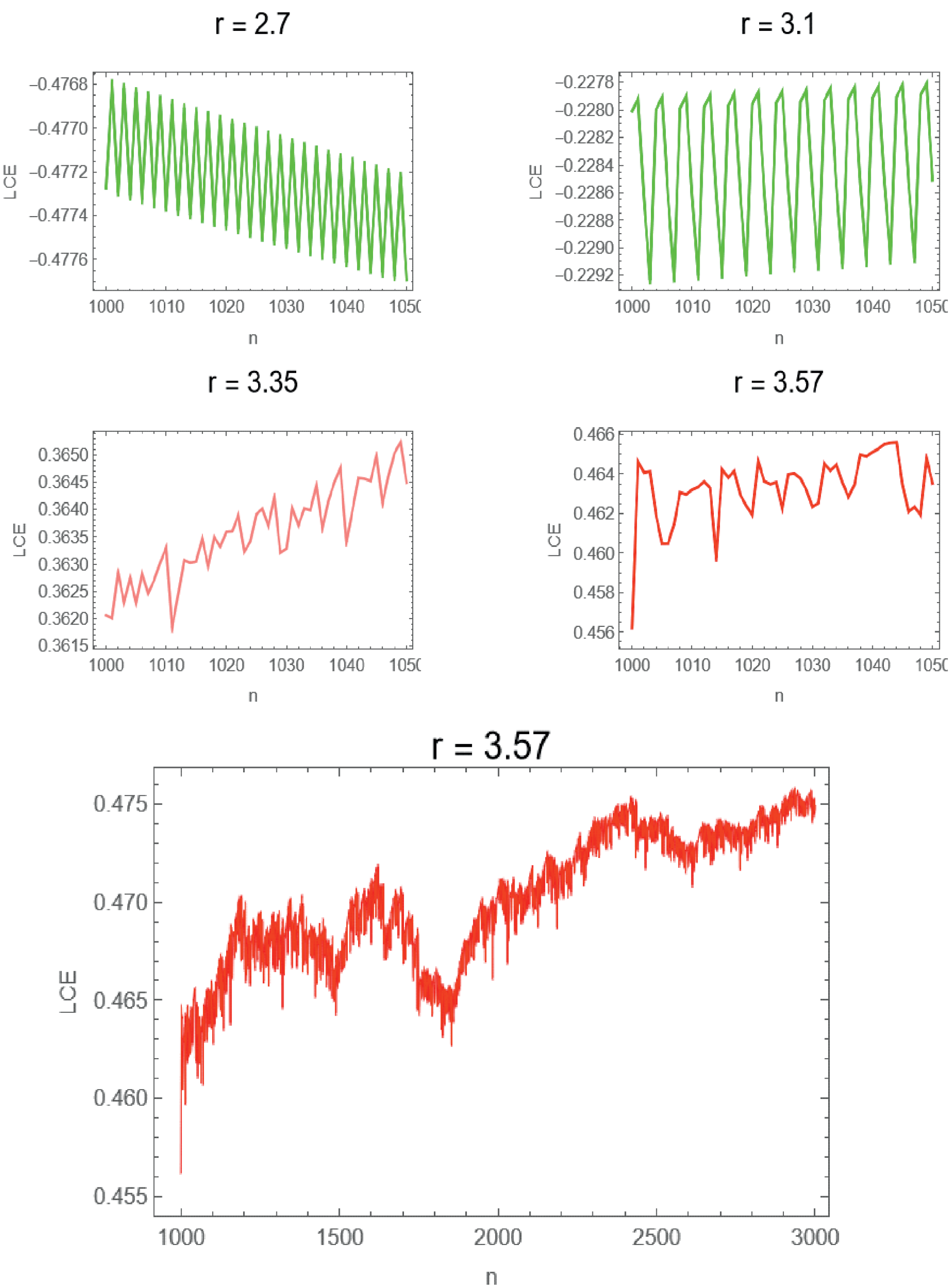

Figure 4: Plots of Lyapunov exponents, (LCEs), of some of the regular and chaotic attractors shown in Figure 3; other parameters are b = 1.5, c = 0.1, k = 0.05

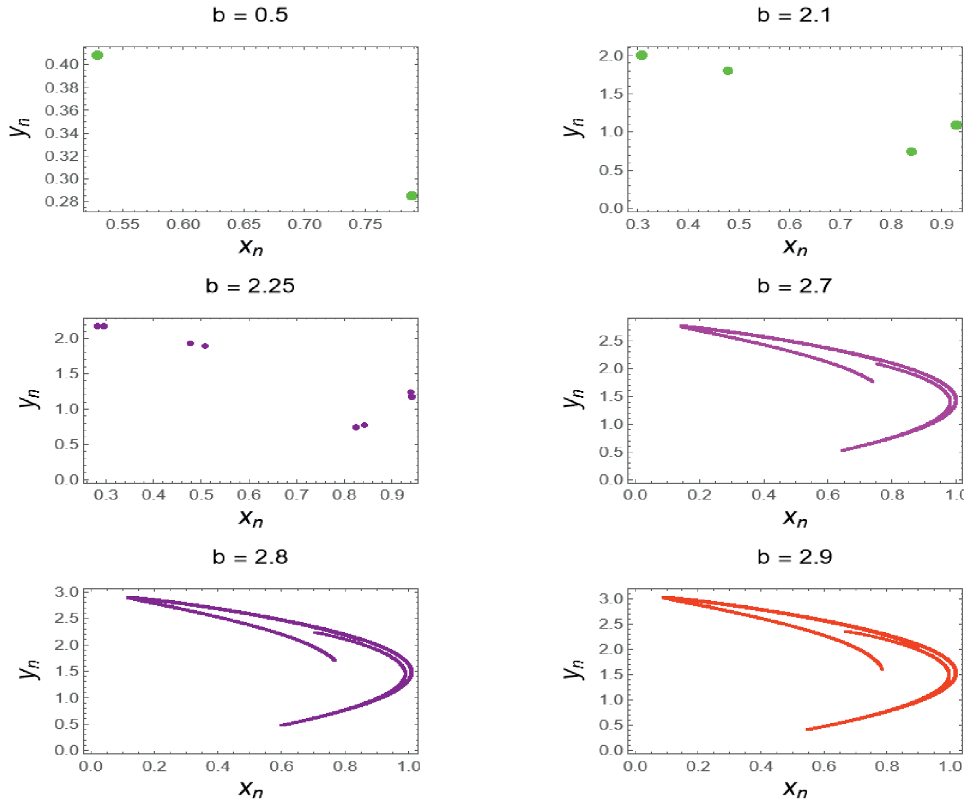

Figure 5: Plots of regular and chaotic attractors of system (2.1) for different values of parameter *b* when other parameters are *r* = 3.0, *c* = 0.1, *k* = 0.05

3c. Topological Entropies

The notion of topological entropy was introduced and suggested that the topological entropy describes measure of the complexity of a dynamical system, [4-5]. It is calculated statistically by using the law of probability. Actually, the topological entropy measures the exponential growth rate of the number of distinguishable orbits as time advances.

Method to measure topological entropy, explained as follows:

Consider a finite partition of a state space *X* denoted by $P = \{A_1, A_2, A_3, \ldots, A_N\}$ Then a measure μ on *X* with total measure μ(*X*) = 1 defines the probability of a given reading as

$$p_i = \mu(A_i), \ i = 1, 2, .., N. \tag{3.1}$$

Then the entropy of the partition be obtained as (Fig. 7). A 3D view is shown in Fig. 8.

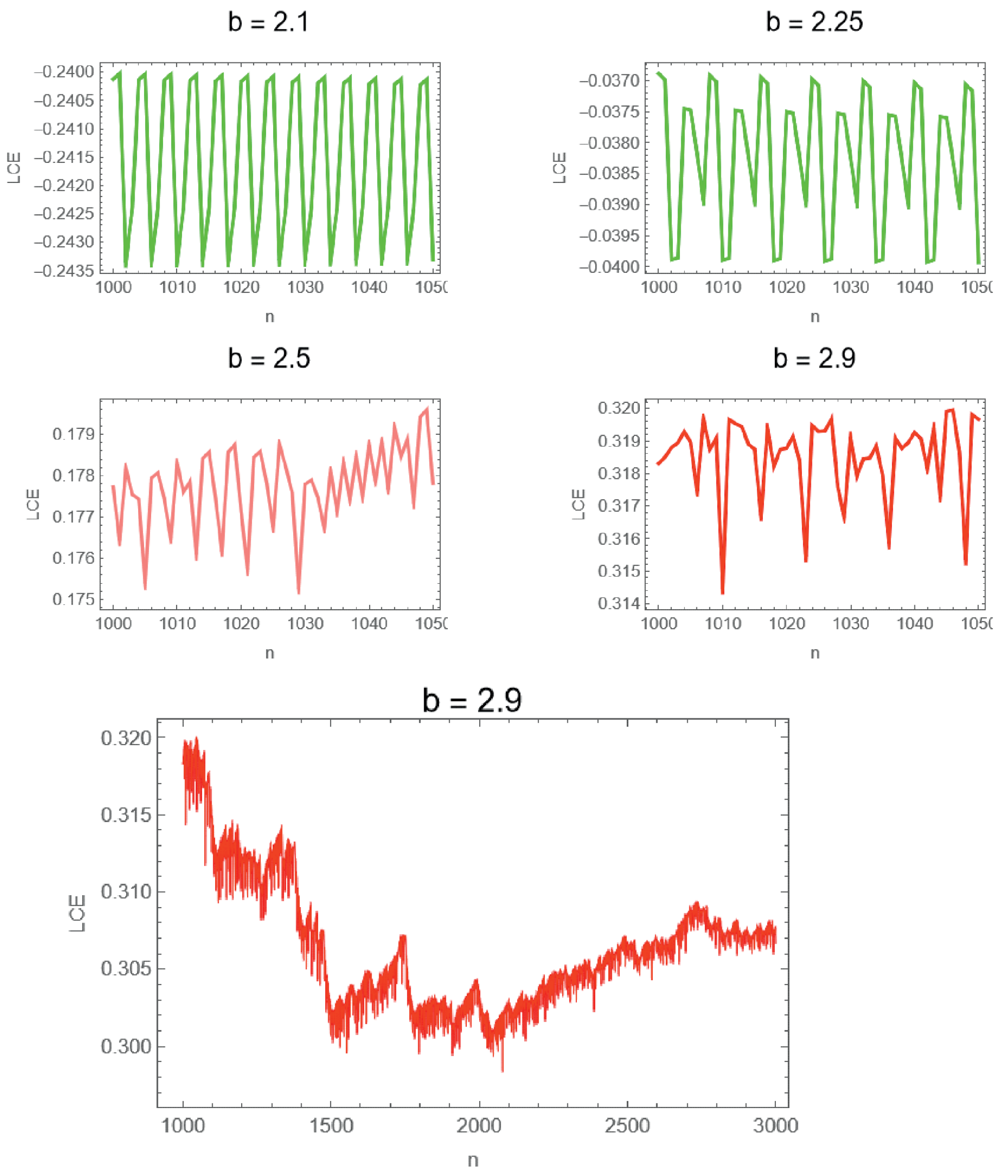

Figure 6: Plots of Lyapunov exponents, (LCEs), for some of the regular and chaotic attractors shown in Figure 5; other parameters are $r = 3.0$, $c = 0.1$, $k = 0.05$

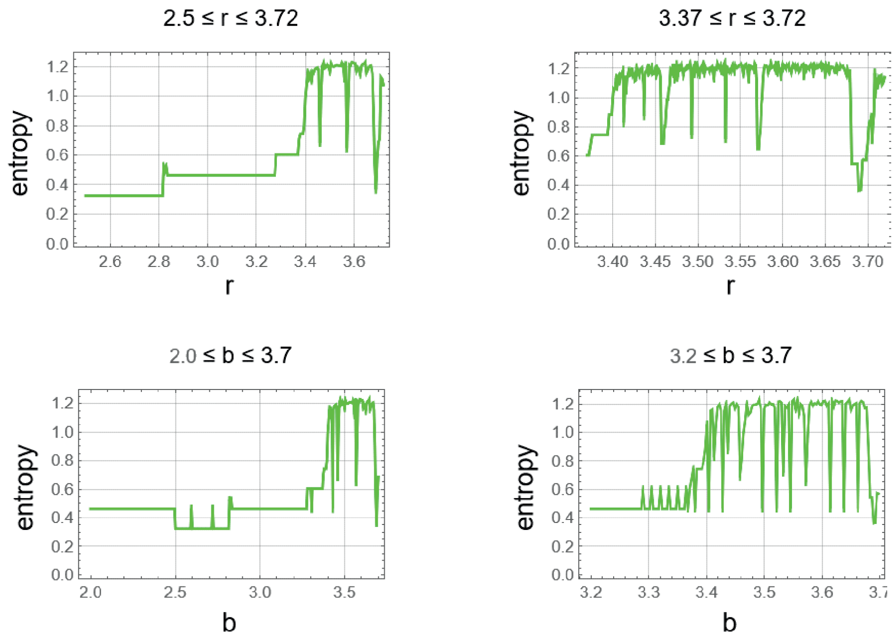

Figure 7: Plots of topological entropies; (i) upper row plots obtained by varying parameter r for b = 1.5, c = 0.1, k = 0.05 and (ii) lower row plots obtained by varying parameter b for r = 3.0 , c = 0.1, k = 0.05. Right column plots displaying fluctuations of topological entropies within periodic windows

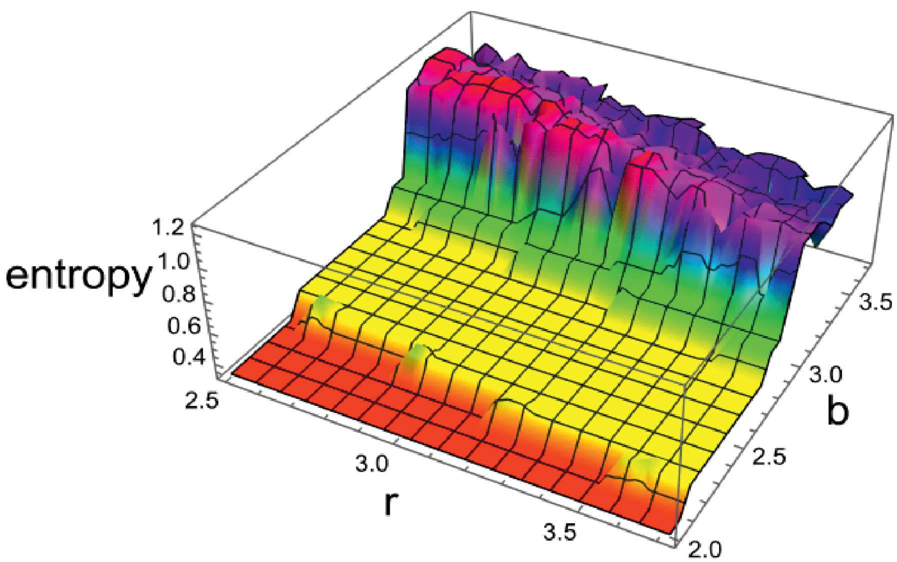

Figure 8: A 3-D plot of topological entropy of system (2.1) for c = 0.1, k = 0.05 and by varying parameters r and b; $2.5 \leq r \leq 3.72$, $2.0 \leq b \leq 3.7$

4. Comments and Discussion

A mathematical model representing climate / environment evolution studied carefully. The model displays the property of complexity shown through bifurcation diagrams, Figure 1 and Figure 2, and plots of topological entropy, Figure 7 and Figure 8, which provides measure of complexity. Plots of topological entropies confirm significant increase in topological entropies and also, fluctuations within the range where periodic windows clearly visible. Complicated periodic windows appearing within bifurcation diagrams show orbits of different period which again splitting and displaying period doubling and chaos adding phenomena. Also, other phenomena like multistability, intermittency etc. may be observed in detailed analysis.

Regular and chaotic climate attractors obtained and shown through Figure 3 and Figure 5 are interesting. Chaotic attractors here appearing like famous Hénon attractor. Formation of Regular and chaotic nature of attractors confirmed by plots of Lyapunov exponents, Figure 4 & Figure 6. Graphical representations of results of this study are interesting and significant.

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